# MATHEMATICS

## RÈSUMÈ OF MATHEMATICS

1. **STANDARD OF PAPER**

The Chief Examiners for Mathematics (Core) and Mathematics (Elective) reported that the standard of the papers compared farourably with that of the previous year.

2. **CANDIDATES' PERFORMANCE**

The Chief Examiners for Mathematics (Core) and Mathematics (Elective) stated that there was a slight decline in performance over that of last year.

3. **A SUMMARY OF CANDIDATES' STRENGTHS**

The Chief Examiners of the two subjects identified the following areas as strengths of candidates this year:

(1) MATHEMATICS (CORE)

Candidates were able to:

a) find the distance between two points when given the coordinates;

b) solve probability problems involving equally likely events;

c) draw a venn diagram from a given information;

d) solve problems involving pie chart;

e) apply Pythagoras' theorem to solve problems;

f) complete tables for multiplication in modulo arithmetic.

(2) MATHEMATICS (ELECTIVE) 2

Candidates exhibited an improvement in:

a) finding the equation of a circle passing through three points;

b) finding the spearman's rank correlation coefficient;

c) constructing cumulative frequency tables and drawing frequency curves;

d) solving problems in series and sequence.

## 4. SUMMARY OF CANDIDATES' WEAKNESSES

(1) MATHEMATICS (CORE)

Candidates showed weaknesses in the following:

(a) showing evidence of reading values from graphs;

(b) translating word problems into mathematical equations;

(c) solving problems on Mensuration and Geometry;

(d) drawing conclusions from logical statements.

(2) MATHEMATICS (ELECTIVE) 2

Candidates showed weaknesses in their inability to:

(a) find the volume of a solid generated using integration;

(b) read correctly the median and the quartiles from the cumulative frequency curve;

(c) solve problems in permutation/combination;

(d) draw a relevant diagram to solve problems on moments.

## 5. SUGGESTED REMEDIES

MATHEMATICS (CORE) 2

(i) Candidates should be advised to read carefully and understand the demands of questions before attempting them.

(ii) Teachers should give candidates more exercises and encourage them to work on their own.

(iii) Teachers should be encouraged to have time for weak students and teach them the basic principles and skills in the topics that they fall short in.

MATHEMATICS (ELECTIVE) 2

The following were recommended to help candidates overcome their weaknesses;

(iv) Teachers should give equal attention to the whole syllabus.

(v) Candidates should be exposed to a lot of questions as exercises.

(vi) Candidates should be encouraged to show all steps when solving problems.

## MATHEMATICS (CORE) 2

**1. STANDARD OF THE PAPER**

The standard of the paper compared favorably with that of the previous year.

**2. PERFORMANCE OF CANDIDATES**

The performance of candidates was quite good but declined compared to the previous year.

**3. SUMMARY OF CANDIDATES' STRENGTHS**

Candidates' strengths were evident in the following areas:

a) find the distance between two points when given the coordinates;

b) solve probability problems involving equally likely events;

c) to draw Venn diagram from a given information;

d) solve problems involving pie chart;

e) apply Pythagoras' theorem to solve problems;

f) complete table for multiplication in modulo arithmetic.

**4. SUMMARY OF CANDIDATES' WEAKNESSES**

Candidates showed weaknesses in the following:

a) showing evidence of reading values from graphs;

b) translating word problems into mathematical equations;

c) solving problems on Mensuration and Geometry;

d) drawing conclusions from logical statements.

**5. SUGGESTED REMEDIES**

(a) Candidates should be advised to read carefully and understand the demands of questions before attempting them.

(b) Teachers should give candidates more exercises and encourage them to work on their own.

(c) Teachers should be encouraged to have time for weak students and teach them the basic principles and skills in the topic that they are weak in.

**6. DETAILED COMMENTS**

**Question 1**

**(a) (i) Draw the multiplication $\otimes$ table in modulo 9 on {2, 3, 7, 8}.**

**(ii) Use the table to find the truth set of $8 \otimes m = 2$.**

**(b) Consider the following statements:**

***X*: All policemen wear uniform;**

***Y*: Civil servants do not wear uniform.**

**If *P* = {policemen}, *T* = {people who wear uniform} and *C* = {civil servants},**

**(i) draw a Venn diagram to illustrate *X* and *Y*;**

(ii) **use the venn diagram to determine which of the following implications are valid or not valid conclusions from *X* and *Y*;**

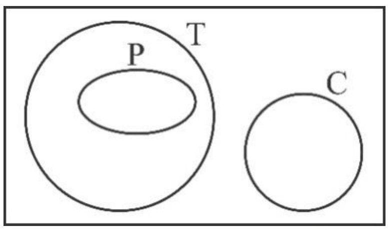
(α) **Adu wears uniform ⟹ Adu is a policeman;**

(β) **Ofei is a policeman ⟹ Ofei is not a Civil Servant;**

Part (a) of this question was well answered by most candidates. They were able to construct the multiplication ⊗ table in modulo 9 on the set given and used it to find the truth set of 8 ⊗ $m$ = 2. A few candidates, however lost the mark for (a)(ii) because they wrote m = 7 instead of {$m$ : $m$ = 7} or {7}

Part (b) was not well answered by most candidates. They could not draw correctly the Venn diagram to illustrate the statements *X* and *Y*.

Candidates were required to answer the Venn diagram in part (b) as follows:




(α) Not valid

(β) Valid

**Question 2**

**(a)** **Given that $p = \frac{q-t}{t-1}$,**

(i) **Make $t$ the subject of the relation.**

(ii) **Find the value of $t$ when $p = \frac{2}{3}$ and $q = \frac{3}{4}$.**

**(b)** **Given that $m = \frac{2x}{1-x^2}$ and $n = \frac{2x}{1+x}$, express, in simplest form, (2$m$ – $n$) in terms of $x$.**

The part (a) of this question was well answered by most candidates. They were able to make $t$ the subject of the relation and went ahead to substitute the values of $p = \frac{2}{3}$ and $q = \frac{3}{4}$ to get value of $t = \frac{1}{20}$

Part (b) of the question was poorly answered by most candidates. Most of them did the substitution correctly but could not find the common denominator to help them do the simplification to come out with the correct answer.

Candidates were required to answer part (b) as follows:

$$2\left(\frac{2x}{1-x^2}\right) - \frac{2x}{1+x}$$

$$= \frac{4x - 2x(1-x)}{(1-x)(1+x)}$$

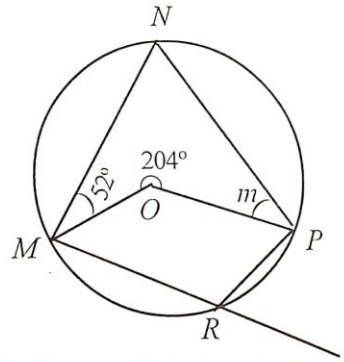
$$= \frac{2x + 2x^2}{(1-x)(1+x)}$$

$$= \frac{2x(1+x)}{(1-x)(1-x)}$$

$$= \frac{2x}{1-x}$$

## **Question 3**

**(a)**




**In the diagram is a circle *MNPR* with centre *O*. The reflex angle at *O* is $204^0$, $\angle NMO = 52^0$. Find the value of *m*.**

**(b) A ladder of length 6.5 m leans against a vertical wall. If the top of the ladder is 3.6 m from the foot of the wall, calculator, correct to two decimal places, the distance from the wall to the foot of the ladder.**

Some of the candidates were able to solve the part (a) correctly. Quite a number of candidates were not able to solve this part correctly. They could not deduce that $\angle MNP = \frac{156}{2} = 78^0$, some could not form the correct equation to get $m = 26^0$ as required. The part (b) of the question was well answered by most candidates.

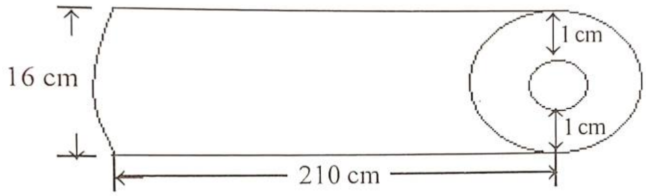
Candidates were required to answer part (a) as follows:

$$\angle MNP = \frac{156}{2}$$
$$= 78^0$$

$$m + 204 + 78 + 52 = 360^0$$
$$m = 360 - 334$$
$$= 26^0$$

**<u>Question 4</u>**

**(a)**




**The diagram is a cylindrical pipe with length 210 cm. It has external diameter of 16 cm and is 1 cm thick. Find the volume of the metal used in constructing the pipe.**
$\left[\boldsymbol{Take\ \pi = \frac{22}{7}}\right]$

**(b) The coordinates of the points *M* and *N* are (5, -7) and (0, 5), respectively. Calculate the distance between *M* and *N*.**

The part (a) of this question was poorly answered by most candidates. They were not able to find the external radius of the cylinder to be $\frac{16}{2}$ = 8 cm and the radius of the internal cylinder to be $\frac{14}{2}$ = 7cm. This affected the volume of the external and internal cylinders respectively which gave them a wrong answer. The part (b) was well answered by most candidates.

Candidates were required to answer part (a) as follows:

Volume of external cylinder = $\frac{22}{7} \times 8 \times 8 \times 210$
= 42, 240 $cm^3$

Volume of internal cylinder = $\frac{22}{7} \times 7 \times 7 \times 210$
= 32, 340$cm^3$

Volume of metal used = 42240 – 32,340
= 9,900$cm^3$

**<u>Question 5</u>**

**Two dice are thrown together once. Find the probability of obtaining:**

**(a) an odd number on the first die and 6 on the second;**

**(b) a number greater than 4 on each dice;**

**(c) a total of 9 or 11.**

This question was well answered by most candidates. They were able to draw a table for outcomes of the two dice and used the table to answer (a), (b) and (c) correctly. A few candidates however could not answer the question correctly.

**<u>Question 6</u>**

**In a military camp, 50 officers had a choice of beans, plantain and rice. Of these officers, 21 chose beans, 24 plantains and 18 rice. Also 3 chose beans only, 9 plantain only, 2 rice only and 5 chose all three kinds of food.**

**(a)** **Illustrate the information on a Venn diagram.**

**(b)** **Use the Venn diagram to find the number of officers who chose:**

**(i)** **plantain and beans only;**

**(ii)** **exactly two kinds of food;**

**(i)** **none of the three kinds of food.**

A very popular question which was well answered by most candidates. They were able to illustrate the information given correctly on the Venn diagram, and used the Venn diagram to come out with the three equations required to helped them answer the part (b) correctly. Quite a number of candidates, however, were not able to solve the question correctly.

**<u>Question 7</u>**

**(a)** **Copy and complete the table of values for the relation $y = 4 + 3x - 2x^2$, for $-4 \leq x \leq 4$.**

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | -10 | | 4 | | 2 | | |

**(b)** **Using a scale of 2 cm to 1 unit on the $x$ – axis and 2 cm to 5 units on the $y$ – axis, draw the graph for the relation $y = 4 + 3x - 2x^2$, for $-4 \leq x \leq 4$.**

**(c)** **Using the graph, find the:**

**(i)** **equation of line of symmetry of the curve;**

**(ii)** **maximum point of the curve;**

**(iii)** **values of $x$ for which $y$ decreases as $x$ increases.**

This question was well answered by most candidates. Most candidates were able to complete the table correctly and used it to draw the quadratic graph. The candidates then used the quadratic graph to answer the part (c) correctly.

Quite a number of candidates could not draw the quadratic graph correctly and as such could not answer the follow-up questions in part (c). They used wrong scales instead of the scale of 2 cm to 1 units on the x-axis and 2cm to 5 units on the y-axis given to draw the graph.

**<u>Question 8</u>**

(a) **A solid metal cylinder of height 6 m and diameter 28 cm is melted and recast into smaller solid cylinders. Each of the smaller cylinders is 14 cm high and 0.5 cm in diameter. How many smaller cylinders were obtained? $\left[Take\ \pi = \frac{22}{7}\right]$**

**(b)** **The unit digit of a two digit number is 1 less than the tens digit. If the number is increased by 8 and then divided by the sum of the digits, the results is 8. Find the number.**

This question was poorly answered by most of the candidates. For the part (a), most of the candidates could not find the volume of the big and the smaller cylinders and this affected their answers in finding the number of small cylinders required. Most candidates could not notice the difference in units (cm, m) did not convert them and this affected their answers. There was, however, some good answers from a few candidates on this part of the question.

The part (b) most candidates could not apply the place value concept in numerals to answer the questions. They could not form the correct equations.
Candidates were required to answer part (a) and (b) as follows:
(a)

$$V_{big} = \pi \times 14 \times 14 \times 600$$
$$= 117600\pi cm^3 \text{ or } 369{,}000cm^3$$
$$V_{small} = \pi \times 0.25 \times 0.25 \times 14$$
$$= 0.875\pi cm^3 \text{ or } 2.75\ cm^3$$

Number of small cylinder = $\frac{117{,}600\pi}{0.875\pi}$

$= 134{,}400$

(b)
$x$ – tens digit and $y$ – unit digit

$x = y + 1$ _______(1)

$\frac{(10x+y)+8}{x+y} = 8$

$10x + y + 8 = 8x + 8y$

$2x - 7y = -8$__________(2)

$x = 3, y = 2$

Number = 32

**Question 9**

**(a) The ratio of cars to motorcycles sold at a garage is 5 : 7. If a dealer sold 142 more motorcycles than cars in a particular month, find the number of each type of vehicle sold.**

**(b) The probabilities of an athlete winning two independent events are $\frac{3}{5}$ and $\frac{2}{9}$. Find the probabilities of winning:**
**(i) only one event;**
**(ii) none of the events**

This was a popular question and was well answered by most candidates. For the part (a), most candidates were able to come up with the respective equations required, solved them correctly; the number of cars = 355 and the number of motorcycles = 497.
The part (b) was also well answered by most candidates. A few candidates however could not answer the question correctly.
Candidates were required to answer part (b) as follows:

$P$(only one event) = $\left(\frac{3}{5} \times \frac{7}{9}\right) + \left(\frac{2}{5} \times \frac{2}{9}\right)$

$= \frac{21}{45} + \frac{4}{45}$

$= \frac{5}{9}$

P(none of events) = $\frac{2}{5} \times \frac{7}{9}$

$= \frac{14}{45}$

## Question 10

a) **A cyclist sets out from a town *P* on the bearing of $060^0$ to a town *Q*, 5 km away. He then moves on the bearing of $345^0$ to a town *R*, 6 km from *Q*.**

   **(i) Represent the information on a diagram.**

   **(ii) Calculate, correct to two decimal places**

   **(α) $|PR|$;**

   **(β) the bearing of *R* from *P*.**

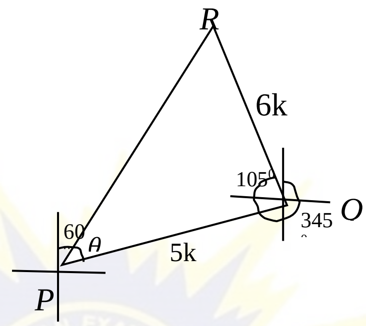
**(b) The following are arranged in order of size: ($x$ – 2), 4, ($x$ + 2), (2$x$ + 1) and 9. If the median is equal to the mean, find the value of $x$.**

The (a)(i) was not correctly answered by most candidates. However, a few a candidates were able to illustrate the information given correctly.

In the part (b), most of the candidates were able to use the information given to find the value of $x = 4$ correctly.

Candidates were required to answer part (a) and (b) as follows:

(a)




$|PR|^2 = 5^2 + 6^2 - 2(6)(5)\cos 105$

$= 61 + 15.529$

$PR = \sqrt{76.529} = 8.75$ km

$\frac{6}{\sin\theta} = \frac{8.75}{\sin 105}$

$\sin\theta = 0.6625$

$\theta = 41.49$

Bearing = 60 – 41.49

= $18.5^0$

(b)

Mean = $\frac{x-2+x+2+4+2x+1+9}{5}$

$= \frac{4x+14}{5} = x + 2$

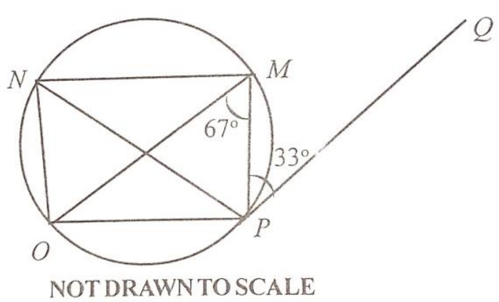
$4x + 14 = 5x + 10$

$x = 4$

## Question 11

**(a) The average mass of refuse generated in a community in a week contains 35% plastic, 27.5% liquid, 17.5% metal, 12.5% vegetable matter and others 7.5%. Draw a pie chart to illustrate the information.**

**(b)**




NOT DRAWN TO SCALE

**In the diagram $\overline{PQ}$ touches the circle $MNOP$ at $P$ and $\overline{NP}$ is a diameter. $\angle MPQ = 33^0$ and $\angle PMO = 67^0$. Find:**

**(i) $\angle MNO$;**

(ii) $\angle MPO$.

The part (a) of this question was correctly answered by most of the candidates. They were able to calculate sectorial angles of the given values correctly, and used them to draw the pie-chart correctly. Part (b) of this question was poorly answered by most candidates confirming students' dislike for circle theorem problems. There were however, some good number of candidates who answered the part (b) of the question.

Candidates were required to answer part (b) as follows:

$\angle ONP = 67^0$

$\angle MNP = \angle MOP = 33^0$

$\angle MNO = \angle MNP + \angle PNO$

$= 33 + 67$

$= 100^0$

In $\Delta MPN$

$\angle MOP = 33^0$

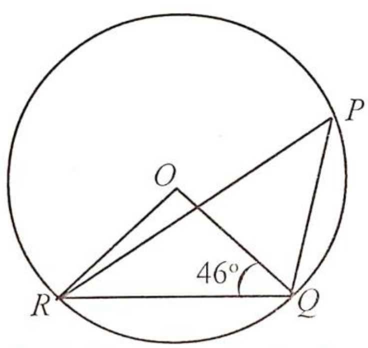
$\angle MPO + 33 + 67 = 180^0$

$\angle MOP = 180 - 100$

$\angle MPO = 80^0$

**<u>Question 12</u>**

**(a)**




NOT DRAWN TO SCALE

**In the diagram, *PQR* is a circle with centre *O*. If $\angle RQO = 46^0$, find $\angle RPQ$.**

**(b) A train is scheduled to cover a distance of 120 km at a certain average speed, r. Due to some technical challenges the average speed is reduced by 5 km/h thereby increasing the scheduled time by 20 minutes. Find the average speed, *r*.**

The part (a) of the question was answered by most candidates. Performance was above average. The part (b) was answered correctly by most of the candidates. They were able to form the necessary equations, and came up with the necessary equations, and came out with the difference in time as

$$\frac{120}{r-5} - \frac{120}{r} = \frac{1}{3}$$

Candidates then cleared fractions and came out with the correct quadratic equation $r^2 - 5r - 1800 = 0$ from which average speed (r) = 45 km/h was obtained. A few candidates however, could not get the correct quadratic equation and this affected their answers.

**Question 13**

**(a)** **A function $g$ is defined on the set of real numbers, $R$, by $g(x) = \frac{1+3x}{x-1}$, $x \neq 1$. Find the:**

**(i)** **image of - 1 under the function g;**

**(ii)** **value for $x$ for which $g(x) = 7$**

**(iii)** **value of $g(-3)$**

**(b)** **The sum of the first ten terms of an Arithmetic Progression (A.P.) is 120. If the fifth term is 6 less than the eighth term, find the:**

**(i)** **common difference;**

**(ii)** **first term.**

Part (a) of this question was well answered by most candidates.
The part (b) was however poorly answered by most candidates. Some of the candidates were able to only get the common difference d = 2 correctly.

Candidates were required to answer part (b) as follows:

$$a + 4d = a + 7d - 6$$

$$3d = 6$$

$$d = 2$$

$$5(2a + (10 - 1)2) = 120$$

$$5(2a + 18) = 120$$

$$10a + 90 = 120$$

$$10a = 30$$

$$a = 3$$

## MATHEMATICS (ELECTIVE) 2

1. **STANDARD OF THE PAPER**

The Chief Examiner for Mathematics (Elective) reported that the standard of the paper was good and compared favourably with that of the previous years.

2. **CANDIDATES' PERFORMANCE**

The Chief Examiner stated that there was a slight decline in performance compared to that of the previous years.

3. **SUMMARY OF CANDIDATES' STRENGTHS**

Candidates showed strength in the following areas:

(a) calculation of rank correlation coefficient;
(b) calculating terms in series and sequences;
(c) finding the equation of a circle passing through three points;
(d) construction of the cumulative frequency tables and drawing the cumulative frequency curve (ogive).

4. **SUMMARY OF CANDIDATES' WEAKNESSES**

The weaknesses of candidates were evident in their inability to:

(a) find the volume of a solid generated using integration;
(b) read correctly the median and the quartiles from the cumulative frequency curve;
(c) find the right technique to handle problems in permutation/combination;
(d) draw correct diagrams as an aid to answering questions involving moments.

5. **SUGGESTED REMEDIES FOR THE WEAKNESSES**

(a) Teachers should give equal attention to the whole topics in the syllabus.
(b) Candidates should be exposed to a lot of questions as exercises. Teachers should lay emphasis on reading values from ogive.
(c) Candidates should be encouraged to show all steps in their workings when solving a problem and not use calculator to jump steps.
(d) learn how to translate word-problems on moment into diagrams to aid in solving them.

6. **DETAILED COMMENTS**

**Question 1**

**Given that one of the roots of $(x - 3)(x - 5) = p$ is three times the other, find the value of $p$.**

The question involved knowledge of the roots of quadratic equations. Most candidates answered the question correctly. Candidates' performance was very good. Candidates were expected to solve the question as:

$(x-3)(x-5) = p$
$x^2 - 8x + (15 - p) = 0$
$\alpha + \beta = 8$…..(1)
$\alpha\beta = 15 - p$….(2)
$\alpha = 3\beta$

$3\beta + \beta = 8$
$4\beta = 8$
$\beta = 2$
$\alpha = 3\beta = 3 \times 2 = 6$
$(6)(2) = 15 - p$
$p = 15 - 12 = 3$

**Question 2**

**Find the volume, in cubic units, of the solid generated when the finite region between the curve $y = x - x^2$ and the $x$ – axis is rotated through one revolution about the $x$ – axis. Leave the answer in terms of $\pi$.**

Candidates were required to find the volume of the solid generated by a finite region between a curve and the *x*-axis. The question involved the use of integration. Candidates were expected to evaluate the integration of the curve and the *x*-axis and use the points or *x*-values for the limits of integration. Most candidates did not attempt the question. Candidates who attempted this question performed below average.

Candidates were expected to solve the question as follows:
$x - x^2 = x(1 - x) = 0$
$\therefore x = 0$ or $x = 1$
$v = \int_0^1 \pi y^2 dx = \int_0^1 \pi(x - x^2)^2 dx$
$= \pi \int_0^1 (x^2 - 2x^3 + x^4) dx$
$= \pi \left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1$
$= \pi \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right]$
$= \frac{\pi}{30}$ *cubic unit*

**Question 3**

**(a)** **Solve:** $\sqrt{(x-4)^3} - 125 = 0$.

**(b)** **Given that** $x = \frac{1}{2-\sqrt{3}}$ **and** $= \frac{1}{2+\sqrt{3}}$, **find the value of** $(x^2 + y^2)$.

The part (a) involved the solution of an irrational equation. Some candidates expanded the left-hand side before solving and this made the solution difficult for them. Some of the candidates were able to get the steps correct and obtained the right solution.

The part (b) was on simplification of surds. Most of candidates used the correct approach and had the required solution. Candidates' performances were very good. Candidates were expected to solve the question as follows:
(a) $\sqrt{(x-4)^3} = 125$
$(x-4)^3 = (125)^2 = 5^6$
$(x-4)^3 = (5^2)^3$
$(x-4) = 5^2$
$x - 4 = 25$
$x = 29$

(b) $x^2 + y^2 = \left(\frac{1}{2-\sqrt{3}}\right)^2 + \left(\frac{1}{2+\sqrt{3}}\right)^2$

$$= \frac{1}{4-4\sqrt{3}+3} + \frac{1}{4+4\sqrt{3}+3}$$
$$= \frac{1}{7-4\sqrt{3}} + \frac{1}{7+4\sqrt{3}}$$
$$= \frac{7+4\sqrt{3}+7-4\sqrt{3}}{49-48}$$
$$= 14$$

**Question 4**

**(a) How many three–digit numbers can be formed using the digits 2, 3, 5, 7, 8, if the number is odd and no digit is repeated?**

**(b) Eleven traders, seeking hotel accommodation were informed that there were 3 vacant rooms which could take 5, 4 and 2 people at a time. In how many ways can these traders be accommodated?**

Some candidates were able to solve part (a). Others were trying to use wrong permutation ideas to solve and eventually got it wrong. Most candidates did not attempt this question.

The part (b) called for the use of combination. Few candidates had a challenge. Candidates' performance was quite good.
Candidates were expected to solve as follows:

(a) $\boxed{4}\boxed{3}\boxed{3}$

$$= 4 \times 3 \times 3$$
$$= 36\ ways$$

(b) $\binom{11}{5} \times \binom{6}{4} \times \binom{2}{2}$

$$= 462 \times 15 \times 1$$
$$= 6930\ ways$$

**Question 5**

**The table shows the marks obtained by seven students in Physics and Mathematics tests.**

| ***Students*** | **A** | **B** | **C** | **D** | **E** | **F** | **G** |
|---|---|---|---|---|---|---|---|
| ***Physics*** | **13** | **15** | **16** | **17** | **14** | **12** | **11** |
| ***Mathematics*** | **12** | **15** | **17** | **16** | **18** | **11** | **14** |

**Calculate the rank correlation coefficient between marks obtained in Physics and Mathematics.**
Most of the candidates were able to solve the question correctly to arrive at the required solution.

The formula for the spearman's rank correlation coefficient was correctly used by candidates.
Candidates' performance was very good.

Candidates were expected to solve the question as follows:

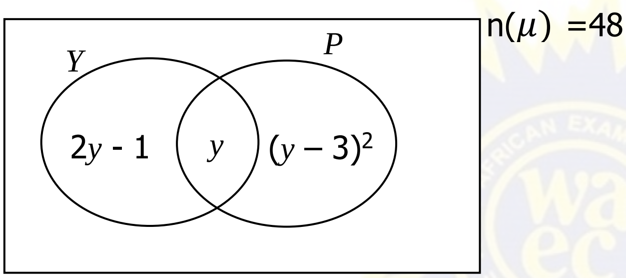
| Student | $R_P$ | $R_M$ | $d_{PM}$ | $d^2{}_{PM}$ |
|---|---|---|---|---|
| A | 3 | 2 | 1 | 1 |
| B | 5 | 4 | 1 | 1 |
| C | 6 | 6 | 0 | 0 |
| D | 7 | 5 | 2 | 4 |
| E | 4 | 7 | -3 | 9 |
| F | 2 | 1 | 1 | 1 |
| G | 1 | 3 | -2 | 4 |
| | | | $\sum d_{PM}{}^2 = 20$ | |

Rank correlation,

$$r_{PM} = 1 - \frac{6\sum d_i{}^2}{n(n^2-1)}$$
$$= 1 - \frac{6\times 20}{7\times(7^2-1)}$$
$$= 1 - 0.3571$$
$$= 0.6429$$

**Question 6**

1. **The breakfast menu in a hotel is a choice of Yam (*Y*), Plantain (*P*) or both. The Venn diagram shows the choices made by 48 guests of the hotel.**




**(a) Find the value of *y*.**

**(b) What is the probability that a guest selected at random chooses either Yam or Plantain but not both?**

Candidates were to calculate *y* and use it to solve for the part (b) of the question. The question was well solved by most candidates. Candidates' performance was very good.
Candidates were expected to solve the question as follows:

(a)

$$2y - 1 + y + (y-3)^2 = 48$$
$$3y - 1 + y^2 - 6y + 9 = 48$$

$$y^2 - 3y - 40 = 0$$
$$(y-8)(y+5) = 0$$
$$y = 8$$

( b)

$$P(Y\ or\ P) = \frac{n(only\ Y) + n(only\ P)}{Total}$$
$$= \frac{2(8) - 1 + (8-3)^2}{48}$$
$$= \frac{15+25}{48}$$
$$= \frac{40}{48} = \frac{5}{6} \text{ or } 0.8333$$

**<u>Question 7</u>**

**A particle with a uniform acceleration of 6ms$^{-2}$ covers 45 m in the 6$^{th}$ second of its motion. Find its initial velocity**.

Candidates were required to find the initial velocity of a particle in motion. Most of the candidates did not get the idea that it was the line interval between 5s and 6s that was to be used. Most candidates could not solve the problem. Candidates' performance was below average.

Candidates were expected to solve the question as follows:

$a = 6\ ms^{-2},\ S = 45\ m, t = 6secs$

$S = ut + \frac{1}{2}at^2$

$S_2 - S_1 = 45\ m$

$S_1 = 5u + \frac{1}{2} \times 6 \times 25$…..(1)

$S_2 = 6u + \frac{1}{2} \times 6 \times 36$…..(2)

$45 = u + 3(36 - 25)$

$45 = u + 33$

$u = 45 - 33$

$u = 12ms^{-1}$

**<u>Question 8</u>**

**The magnitude of a force (*pi* + 20*j*) is 29 *N*. Find the:**

**(a) possible values of *p*;**

**(b) direction of the force, correct to the nearest degree.**

Candidates were required to calculate the possible values of a force *p* and the direction of *p*. The possible values of *p* were well calculated. Candidates were able to calculate the angle of *p* to the vertical but had problem representing the direction as 046$^0$ or N46$^0$E.

Almost all candidates had a problem calculating the direction of the other force: -21*i* + 20*j* which was 314$^0$ or *N*46$^0$*W*. Candidates' performance for the question was average. Candidates were expected to solve the question as follows:

(a) $F = Pi + 20j$

$|F| = \sqrt{P^2 + 20^2} = 29$

$P^2 + 400 = 29^2 = 841$

$P^2 = 441$

$P = \pm\sqrt{441} = \pm\ 21$

(b) $F = 21i + 20j$ or $F = -21i + 20j$

Direction of $21i + 20j$

$tan\theta = \frac{20}{21}$

$\theta = tan^{-1}\left(\frac{20}{21}\right)$

$\theta = 43.602°$

$\therefore Direction = 046°\ or\ N46°E$

Direction of $-21i + 20j$

$tan\theta = \frac{-20}{21}$

$\theta = tan^{-1}\left(\frac{-20}{21}\right)$

$\theta = 43.6028°$

$\therefore Direction = 314°\ or\ N46°W$

**Question 9**

**An exponential sequence (*G. P.*) has a positive common ratio. If the sum to infinity of the sequence is 25 and the sum of the first 2 terms is 16, find the:**

**(a) fifth term;**

**(b) sum of the first 4 terms of the sequence.**

The question was on exponential sequence. Candidates were required to find the first term and the sum of the first four (4) terms of the sequence. Most of the candidates answered the question. Candidates' performance was very good. Candidates were able to find the common ratio (*r*) and the first term (*a*).

$S_\infty = \frac{a}{1-r} = 25$

$a + ar = 16$

$a(1+r) = 16$

$a = \frac{16}{1+r}$

$25 = \frac{\left(\frac{16}{1+r}\right)}{1-r}$

$25 = \frac{16}{(1-r)(1+r)} \Longrightarrow 25 = \frac{16}{1-r^2}$

$1 - r^2 = \frac{16}{25}$

$1 - \frac{16}{25} = r^2$

$\frac{9}{25} = r^2$

$\therefore r = \frac{3}{5}$

$a = \frac{16}{1+\frac{3}{5}}$

$= 10$

$U_5 = 10\left(\frac{3}{5}\right)^4 = 10 \times \frac{81}{625}$

$= 1\frac{37}{125}$ *or* $1.296$

(b) $S_4 = \frac{10\left(1-\left(\frac{3}{5}\right)^4\right)}{1-\frac{3}{5}}$

$= \frac{10\left(\frac{544}{625}\right)}{\frac{2}{5}}$

$= 21.76$ or $21\frac{19}{25}$

**Question 10**

**The matrices of the Linear transformations *P* and *Q* are given by *P* = $\begin{bmatrix} \mathbf{3} & \mathbf{-2} \\ \mathbf{-1} & \mathbf{4} \end{bmatrix}$ and *Q* = $\begin{bmatrix} \mathbf{5} & \mathbf{2} \\ \mathbf{7} & \mathbf{4} \end{bmatrix}$.**

**(a) Calculate the image of the point (2, -3) under the transformation *Q* followed by *P*.**

**(b) Find the matrix *R* such that *PR* + $\frac{1}{3}$*Q*$^2$ = I, where I is the 2 × 2 identity matrix.**

In the part (a), some candidates rearranged the order of the transformation, instead of finding *PQ* and using it to find the image, they rather calculated *QP* and used it to find the image. After the correct image was found, some candidates failed to write their answer in the coordinate form. The part (b) was not well attempted by most candidates. Candidates' performance for the question was below average. Candidates were expected to solve the question as:

(a) $PQ = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 23 & 14 \end{pmatrix}$

$\begin{pmatrix} 1 & -2 \\ 23 & 14 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

∴The image of the point $(2, -3)$ under the transformation Q followed by P is $(8, 4)$

(b)

Let $R = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$, $Q^2 = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 39 & 18 \\ 63 & 30 \end{pmatrix}$

$\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 39 & 18 \\ 63 & 30 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$3w - 2y + 13 = 1 \Longrightarrow 3w - 2y = -12$…..(1)

$-w + 4y + 21 = 0 \Longrightarrow -w + 4y = -21$….(2)

From equation (2), $w = 4y + 21$…..(3)

$3(4y + 21) - 2y = -12$

$12y + 63 - 2y = -12$

$10y = -7.5$

$\therefore y = -7.5 \; or -\frac{15}{2}$

$w = 4(-7.5) + 21 = -30 + 21 = -9$

$-x + 4z + 10 = 1 \Longrightarrow -x + 4z = -9$

From equation(5),$x = 4z + 9$

$3(4z + 9) - 2z = -6$ …..(4)

$12z + 27 - 2z = -6$…….(5)

$10z = -33$

$z = -3.3 \; or -3\frac{3}{10}$

$x = 4(-3.3) + 9 = -13.2 + 9 = -4.2$

$R = \begin{pmatrix} -9 & -4.2 \\ -7.5 & -3.3 \end{pmatrix}$

**Question 11**

**A circle passes through points *P*(-2,0), *Q*(1, 0) and *R*(2, -1). Find the:**

**(a) coordinates at the centre;**

**(b) radius;**

(c) **equation of the circle.**

Most of the candidates were able to generate the three equations, solved them correctly and obtained the coordinates of the centre, radius and the equation of the circle. It was answered by most candidates and their performances were very good.

The solution is as follows:

$x^2 + y^2 + 2gx + 2fy + c = 0$
$4 + 0 - 4g + 2f(0) + c = 0$
$-4g + c = -4$……..(1)
$1 + 0 + 2g + 2f(0) + c = 0$
$2g + c = -1$……….(2)
$4 + 1 + 2g(2) + 2f(-1) + c = 0$
$5 + 4g - 2f + c = 0$
$4g - 2f + c = -5$…………(3)
From equation (1) and (2) $-4g + c = -4$
$2g + c = -1, 6g = 3 \implies g = \frac{1}{2}$
$c = -4 + 4g$
$c = -1$
$4\left(\frac{1}{2}\right) - 2f - 2 = -5$
$2 - 2f - 2 = -5 \implies -2f = -5 \implies f = \frac{5}{2}$
$(x-a)^2 + (y-b)^2 = r^2$
$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$
$x^2 + y^2 + 2gx + 2fy + c = 0$
$-2ax = 2gx \implies a = -g = -\frac{1}{2}$
$-2by = 2fy$
$b = -f = -\frac{5}{2}$
$(a, b) = \left(-\frac{1}{2}, -\frac{5}{2}\right)$
$(-g, -f) = \left(-\frac{1}{2}, -\frac{5}{2}\right)$

(b)
$a^2 + b^2 - r^2 = c$
$\left(-\frac{1}{2}\right)^2 + \left(-\frac{5}{2}\right)^2 - r^2 = -2$
$\frac{1}{4} + \frac{25}{4} - r^2 = -2$
$\implies \frac{26+8}{4} = r^2 \implies \frac{34}{4} = r^2$
$\therefore r = \sqrt{\frac{34}{4}} = \frac{\sqrt{34}}{2} = 2.9155$

(c) $x^2 + y^2 + 2\left(\frac{1}{2}\right)x + 2\left(\frac{5}{2}\right)y - 2 = 0$
$x^2 + y^2 + x + 5y - 2 = 0$

**<u>Question 12</u>**

(a) **In an examination, the probabilities of Charles scoring the highest mark in Mathematics, Physics and Chemistry are 0.90, 0.75 and 0.80 respectively. Calculate the probability that he will get the highest mark in:**

**(i)** **all the 3 subjects;**

**(ii)** **at least 2 subjects.**

**(b)** **A committee of 3 members is to be formed from a club consisting of 4 men and 5 women. Find the probability that the committee formed will consist of at least one man.**

Most of the candidates did not attempt this question and the few who attempted performed below average. Candidates were expected to solve the question as follows:

(a)(i) $P(M_a) = 0.9,\ P(P_h) = 0.75,\ P(C_h) = 0.80$

$P(M_A{}^I) = 0.1,\ P(P_h{}^I) = 0.25,\ P(C_h{}^I) = 0.20$

$P(of\ obtaining\ the\ highest\ mark\ in\ all\ the\ 3\ subjects)$

$= 0.9 \times 0.75 \times 0.8 = 0.54$

(ii) $P(of\ getting\ the\ highest\ mark\ in\ at\ least\ 2\ subjects)$

$= P(M_a)P(P_h)P(C_h{}^I) + P(M_a)P(C_h)P(P_h{}^I) + P(M_a{}^I)P(C_h)P(P_h) +$
$P(M_a)P(P_h)P(C_h)$

$= (0.90 \times 0.75 \times 0.2) + (0.9 \times 0.8 \times 0.25) + (0.1 \times 0.75 \times 0.8) + (0.9 \times 0.75 \times 0.8)$

$= 0.135 + 0.18 + 0.006 + 0.54$

$= 0.915$

(b) $P(at\ least\ one\ man) = \frac{\binom{4}{1}\binom{5}{2}+\binom{4}{2}\binom{5}{1}+\binom{4}{3}\binom{5}{0}}{\binom{9}{3}}$

$= \frac{(4\times10)+(6\times5)+(4\times1)}{84}$

$= \frac{74}{84}$

$= \frac{37}{42}\ or\ 0.8810$

**<u>Question 13</u>**

**The table shows the distribution of marks scored by candidates in an Examination.**

| ***Marks* (%)** | **1-10** | **11-20** | **21-30** | **31-40** | **41-50** | **51-60** | **61-70** | **71 -80** | **81-90** | **91-100** |
|---|---|---|---|---|---|---|---|---|---|---|
| ***Frequency*** | **2** | **7** | **8** | **13** | **24** | **30** | **6** | **5** | **3** | **2** |

**(a)** **Construct a cumulative frequency distribution table and use it to draw the cumulative frequency curve (ogive).**

**(b)** **Use your graph to estimate the:**

**(i)** **median mark;**

**(ii)** **inter-quartile range.**

Candidates were able to construct the cumulative frequency table and used it to draw the cumulative frequency curve. However, finding the median, the upper and lower quartiles were a challenge to candidates. Candidates' performances were good.

The solution is as follows:

(a)(i)

| Marks | Less Than | Frequency | Cumulative Frequency |
|---|---|---|---|
| 1 – 10 | 10.5 | 2 | 2 |
| 11 – 20 | 20.5 | 7 | 9 |
| 21 – 30 | 30.5 | 8 | 17 |
| 31 – 40 | 40.5 | 13 | 30 |
| 41 – 50 | 50.5 | 24 | 54 |
| 51 – 60 | 60.5 | 30 | 84 |
| 61 – 70 | 70.5 | 6 | 90 |
| 71 – 80 | 80.5 | 5 | 95 |
| 81 – 90 | 90.5 | 3 | 98 |
| 91 - 100 | 100.5 | 2 | 100 |

See the cumulative frequency curve(Ogive) attached

(b) Median = $\left(\frac{100}{2}\right)^{th}$ $value = 50^{th} value$

$= 48.5 \pm 0.5$

$Q_3 = \left(\frac{300}{4}\right)^{th} value = 75^{th} value$

$= 57.5 \pm 0.5$

$Q_1 = \left(\frac{100}{4}\right)^{th} value = 25^{th} value$

$= 37.5 \pm 0.5$

$Interquartile\ range = Q_3 - Q_1$

$= 57.5 - 37.5$

$= 20.0 \pm 0.5$

QUESTION 13


Cum. freq.
GRAPH
100
90
80
70
60
50
40
30
20
10
0
0.5
10.5
20.5
30.5
40.5
50.5
60.5
70.5
80.5
90.5
100.5
Q1
Q2
Q3
Marks
GOOD BETTER BEST WAEC STANDARD GRAPH ©

## Question 14

**A body at rest is acted upon by two forces $P(50\ N, 030^0)$ and $Q(XN, 120^0)$.**

**(a) Express each force as a column vector.**

**(b) If the magnitude of the resultant force is 65 $N$, calculate the:**

**(i) value of $X$, $X > 0$;**

**(ii) direction of the resultant force, correct to the nearest degree.**

Most of the candidates who attempted the question were able to solve it correctly and the performances were very good. Candidates were expected to provide solution as follows:

(a) $P(50N, 030°)$, $Q(XN, 120°)$

$$P = \begin{pmatrix} 50sin30 \\ 50cos30 \end{pmatrix} \ Q = \begin{pmatrix} Xsin120 \\ Xcos120 \end{pmatrix}$$

Resolving the forces and adding:

$$R = \begin{pmatrix} 50sin30 \\ 50cos30 \end{pmatrix} + \begin{pmatrix} Xsin120 \\ Xcos120 \end{pmatrix}$$

b(i) $$R = \begin{pmatrix} 25 \\ 25\sqrt{3} \end{pmatrix} + \begin{pmatrix} \frac{\sqrt{3}}{2}X \\ -\frac{1}{2}X \end{pmatrix} = \begin{pmatrix} 25 + \frac{\sqrt{3}}{2}X \\ 25\sqrt{3} - \frac{1}{2}X \end{pmatrix}$$

$$R = \sqrt{\left(25 + \frac{\sqrt{3}}{2}X\right)^2 + \left(25\sqrt{3} - \frac{1}{2}X\right)^2}$$

$$R^2 = \left(25 + \frac{\sqrt{3}}{2}X\right)^2 + \left(25\sqrt{3} - \frac{1}{2}X\right)^2$$

$$65^2 = \left(25 + \frac{\sqrt{3}}{2}X\right)^2 + \left(25\sqrt{3} - \frac{1}{2}X\right)^2$$

$$4225 = 625 + 25\sqrt{3}X + \frac{3}{4}X^2 + 1875 - 25\sqrt{3}X + \frac{X^2}{4}$$

$$4225 = 2500 + X^2$$

$$X^2 = 1725$$

$$X = \sqrt{1725}$$

$$X = 41.53\ N$$

(ii) $$R = \begin{pmatrix} 25 + \frac{\sqrt{3}}{2}X \\ 25\sqrt{3} - \frac{1}{2}X \end{pmatrix} = \begin{pmatrix} 25 + 0.8660X \\ 25(1.732) - \frac{1}{2}X \end{pmatrix}$$

$$= \begin{pmatrix} 25 + 0.8660 \times 41.53 \\ 43.3 - \frac{1}{2} \times 41.53 \end{pmatrix} = \begin{pmatrix} 25 + 35.965 \\ 43.3 - 20.765 \end{pmatrix}$$

$$= \begin{pmatrix} 60.965 \\ 22.535 \end{pmatrix}$$

$$tan\alpha = \frac{22.535}{60.965}$$

$$\alpha = tan^{-1}\left(\frac{22.535}{60.965}\right)$$

$$= 69.71°$$

$$Direction = 070°$$

**Question 15**

(a) **A uniform beam *PQ* of length 8 m and mass 10 kg is supported horizontally at the end *P* and at point, *R*, 3 m from *Q*. A boy of mass 20 kg walks along the beam starting from *P*. If the beam is in equilibrium, calculate the reactions at *P* and *R* after he had walked 1.5 m.**

**[Take $g$ = 10 $ms^{-2}$]**

**(b) An object *P* of mass 5 kg is suspended by means of two light inextensible strings *TP* and *QP*. The strings *TP* and *QP* make angles of $60^0$ and $30^0$ with the downward vertical respectively. The magnitudes of the tensions in *TP* and *QP* are $T_1$ and $T_2$ respectively.**

**(i) Illustrate the information in a diagram.**

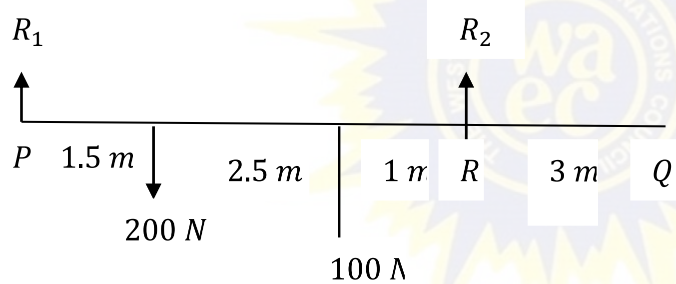
**(ii) Calculate, correct to one decimal place, the values of $T_1$ and $T_2$.**

**[Take $g$ = 10 $ms^{-2}$]**

Candidates could not draw the necessary tree-body diagrams and use it to solve the problem. The part (a) involves the taking of moment of forces to calculate the reaction of the supports *P* and *R*. Candidates had problem solving the question.

The part (b) was well attempted by candidates and candidates' performances on the question was average. Candidates were expected to solve the question as follows:

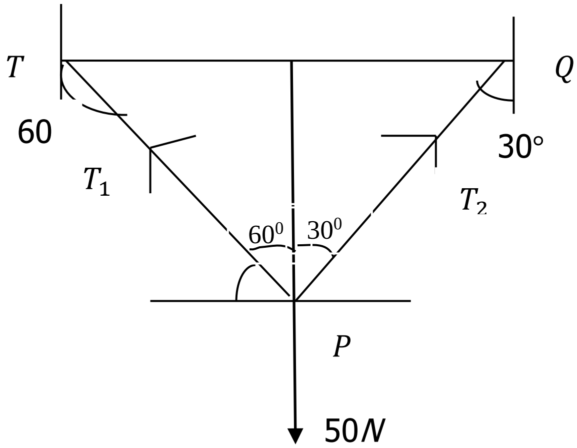
(a




Taking moments about R,

$$R_1 \times 5 = 100 \times 1 + 200 \times 3.5$$
$$= 100 + 700 = 800$$
$$R_1 = \frac{800}{5} = 160\ N$$
$$R_1 + R_2 = 200N + 100N = 300N$$
$$160N + R_2 = 300N$$
$$R_2 = 300 - 160 = 140N$$

(b)




Using Lami's theorem, $\frac{50}{sin90°} = \frac{T_1}{sin150°} = \frac{T_2}{sin120°}$

$$\frac{50}{sin90°} = \frac{T_1}{sin150°}$$

$$T_1 = \frac{50 \times sin150°}{sin90°} = 50\ \times 0.5 = 25.0\ N$$

$$\frac{T_2}{sin120°} = \frac{50}{sin90°}$$

$$T_2 = \frac{50 \times sin120°}{sin90°} = 50\ \times 0.8660 = 43.3N$$